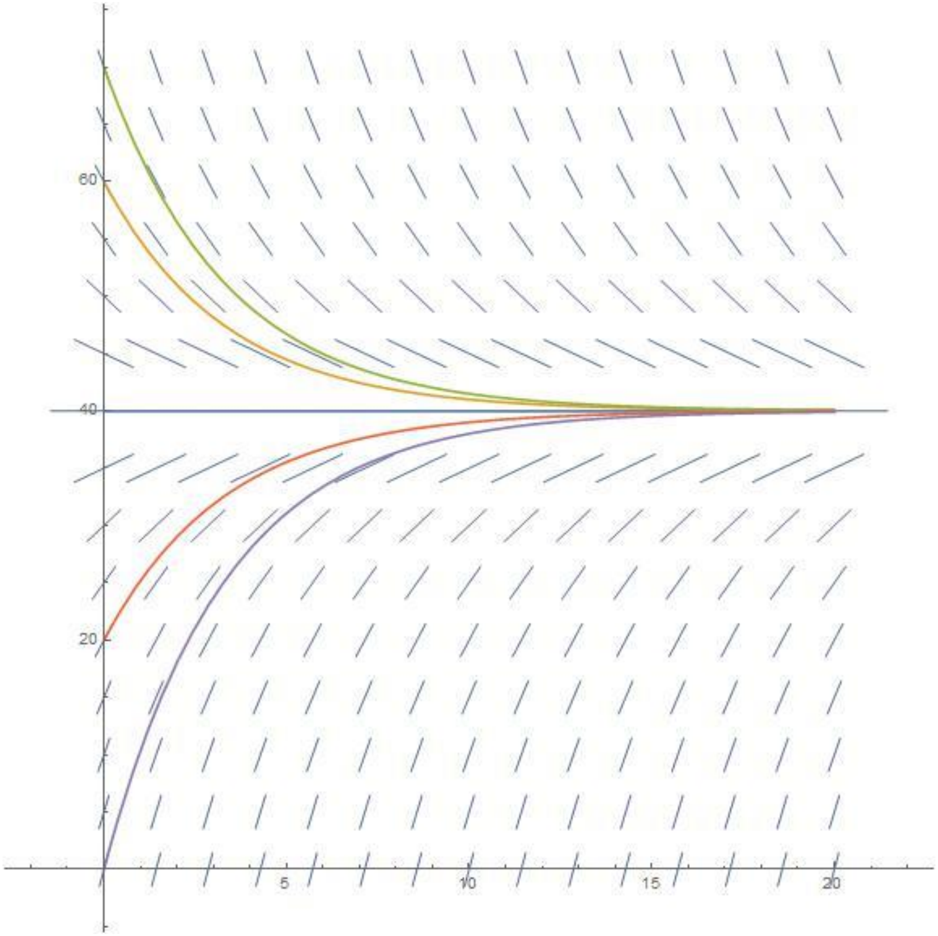
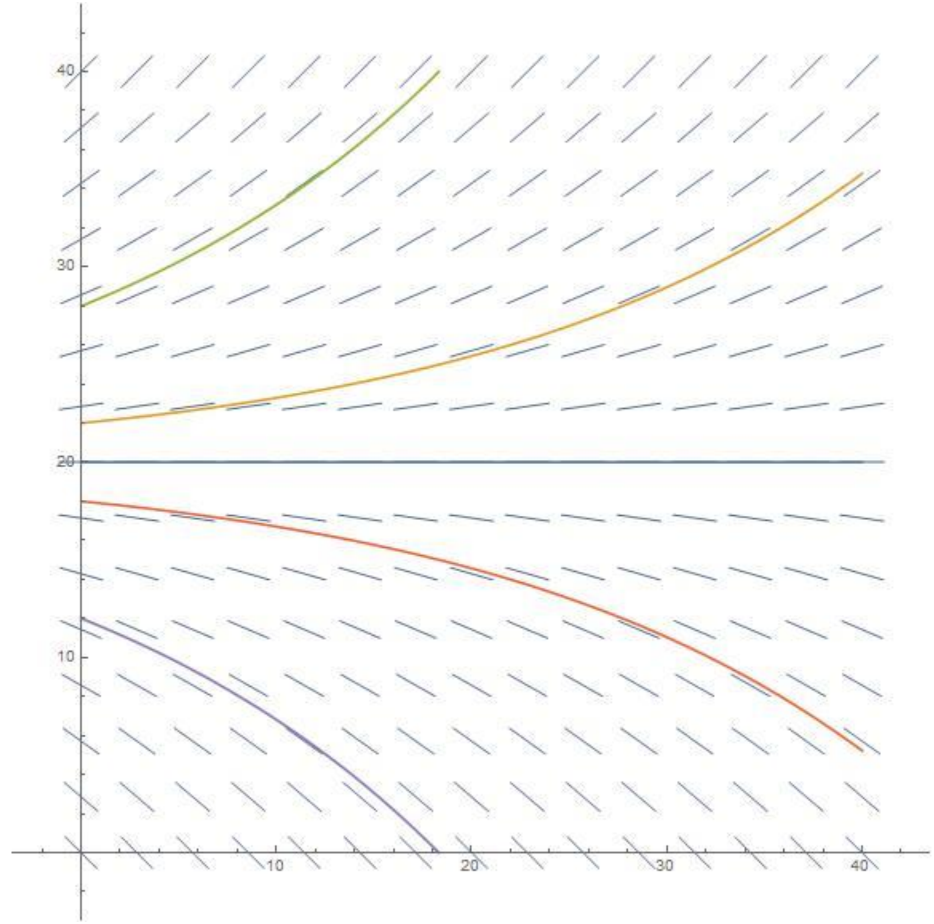
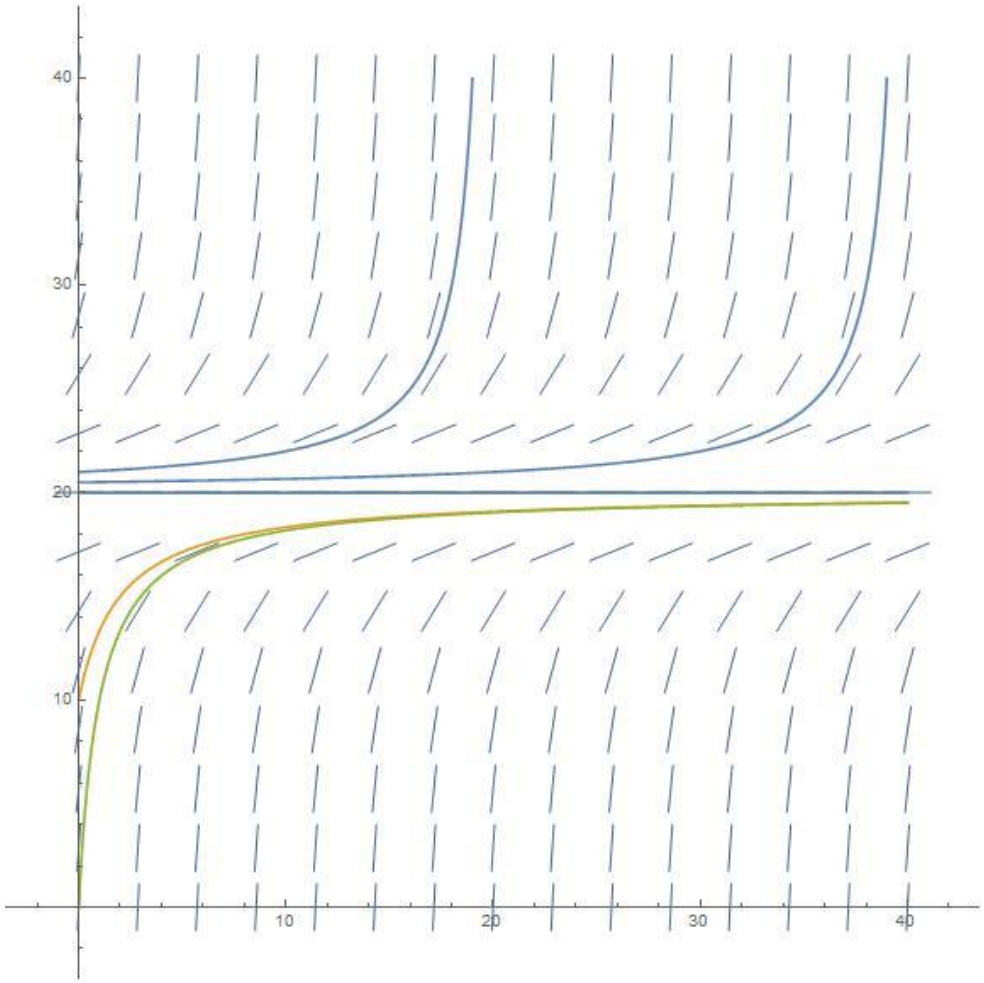


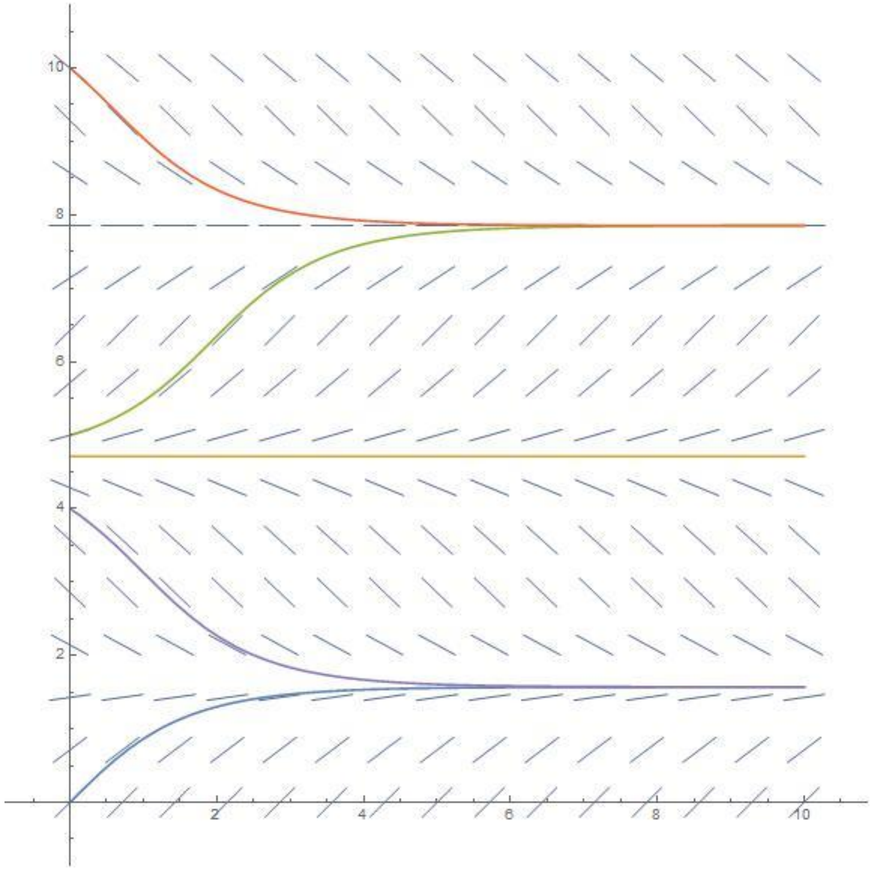
y(t) = 40 is an asymptotically **STABLE** equilibrium sol’n to dy/dt = -0.3(y-40)



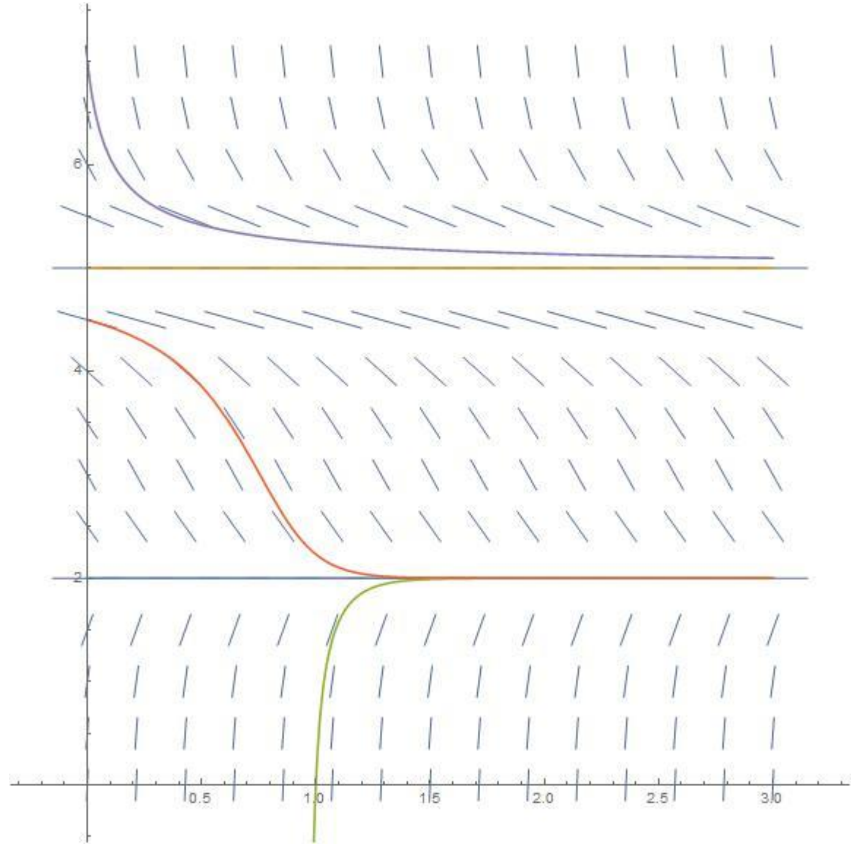
y(t) = 20 is an asymptotically **UNSTABLE** equilibrium sol’n to dy/dt = 0.3(y-20)



y(t) = 20 is an asymptotically **SEMISTABLE** equilibrium sol'n to $dy/dt = 0.05(y-20)^2$



dy/dt = cos(y)



$dy/dt = (2-y)\,(y-5)^2$

Some population models

y(t) = “size of population at time *t*”

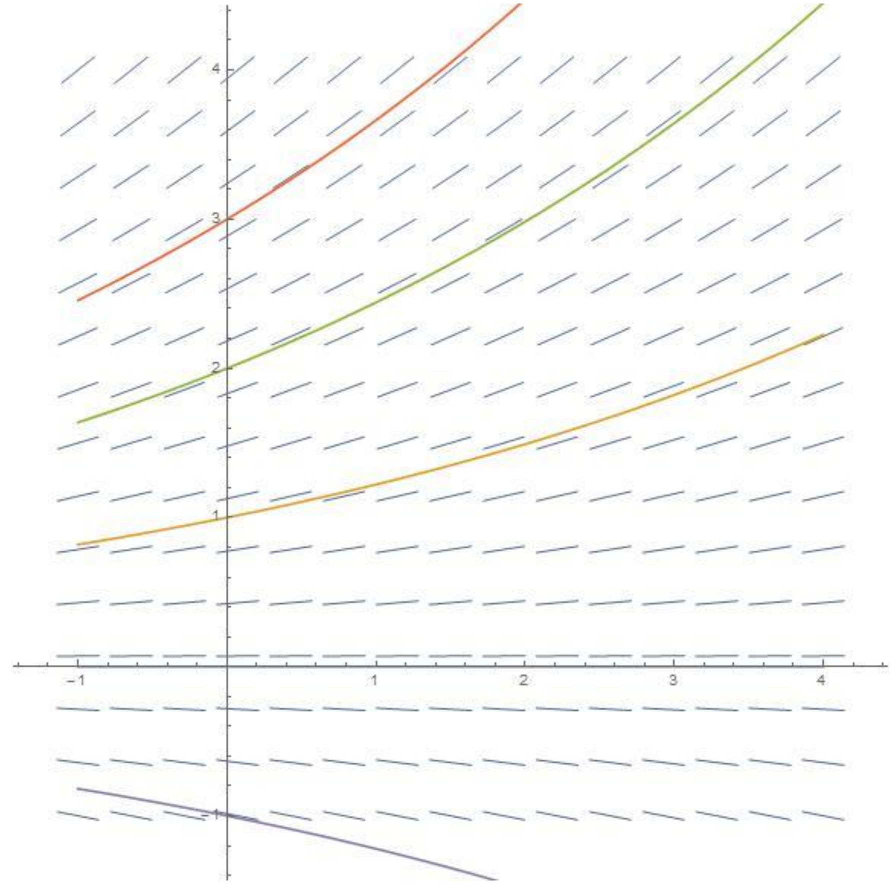
dy/dt = “rate of change” (i.e. number of people per year by which population increases)

Model 1: Unrestricted growth
(or Law of Natural Growth).
“rate is proportional to population size”
r = “prop. constant”

= “relative growth rate”

$$\text{Model: } \frac{dy}{dt} = ry, \qquad y(0) = y_0$$

$$\text{Solution: } y(t) = y_0 e^{rt}$$

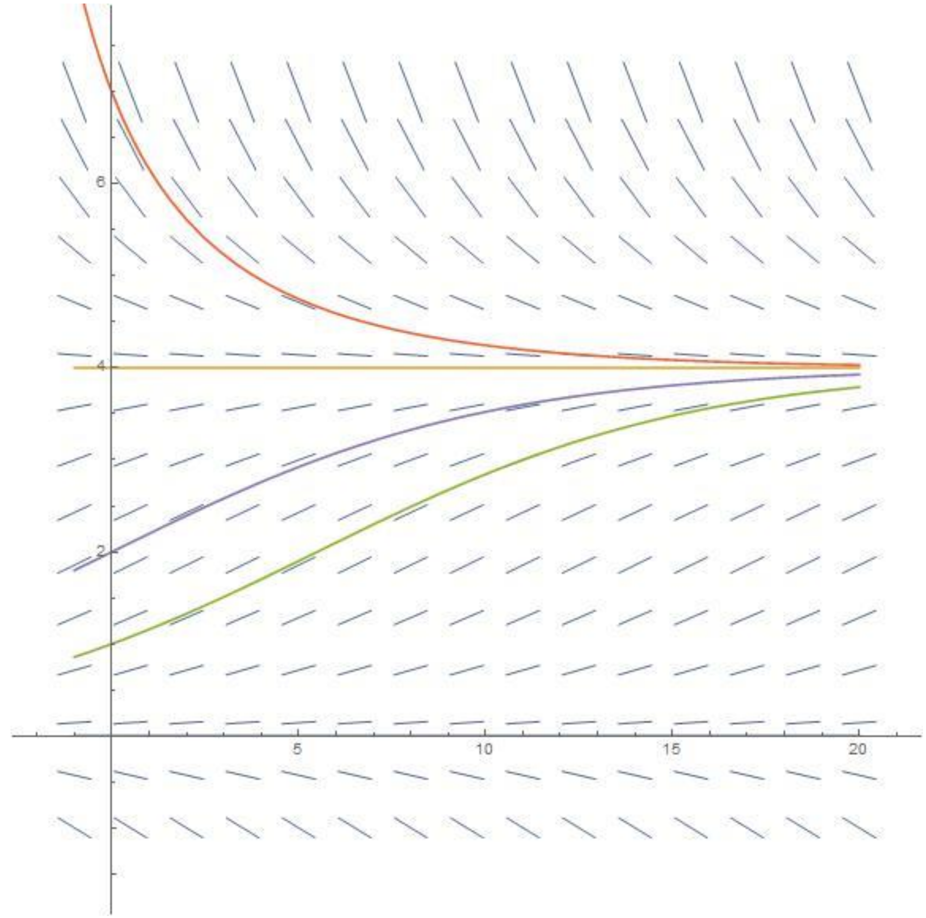


dy/dt = ry, with r > 0

Restricted growth:
There are many models for restricted growth. Most look like

$$\frac{dy}{dt} = h(y)y,$$

where h(y) ≈ r when y is "small",
but as y approaches some capacity C, h(y) goes to zero.



Model 2: The Logistic Equation

r = relative growth rate

K = capacity

$$\text{Model:} \frac{dy}{dt} = r\left(1 - \frac{y}{K}\right)y, \qquad y(0) = y_0$$

dy/dt = r(1-y/K)y, with r, K > 0

$$\text{Solution: } y(t) = \frac{y_0 K}{y_0 + (K - y_0)e^{-rt}}$$

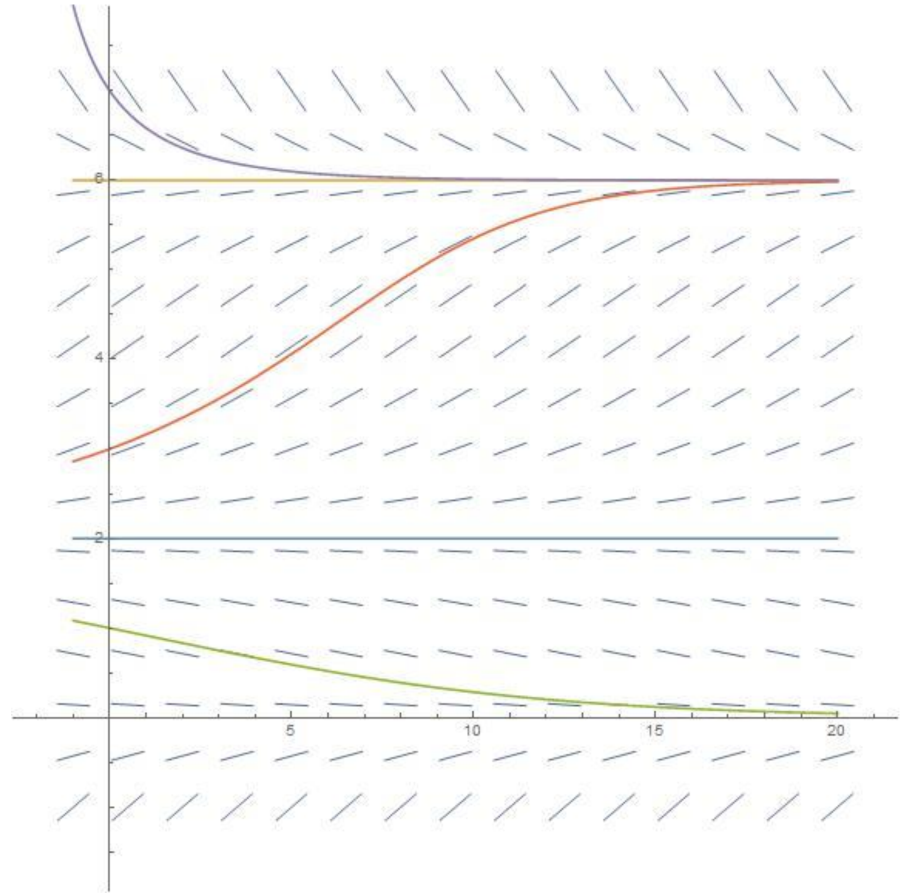
Assume the population dies out if it is below a certain “threshold”, but otherwise behaves like restricted growth. Here is a model for that type of population:

Model 3: Restricted growth with a threshold

r = relative growth rate
K = capacity
T = threshold

Model: $$\frac{dy}{dt} = -r\left(1-\frac{y}{T}\right)\left(1-\frac{y}{K}\right)y$$

Solution: $$\frac{y(y-K)^{T/(K-T)}}{(y-T)^{K/(K-T)}} = De^{-rt}$$



dy/dt = -r(1-y/T)(1-y/K)y,

with r > 0 and K > T > 0.